

Heavy Light Weak Matrix Elements*

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I review the status of lattice calculations of heavy-light weak matrix elements, concentrating on semileptonic B decays to light mesons, $\bar{B} \rightarrow K^* \gamma$, the B meson decay constant, f_B , and the mixing parameter B_B .

1. INTRODUCTION

In this review I will focus on calculations of selected matrix elements involving b quarks. The semileptonic decays $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ and $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ depend on the CKM element V_{ub} which fixes one side of the unitarity triangle. Lattice calculations open the possibility for a model independent extraction of this quantity. The rare radiative decay $\bar{B} \rightarrow K^* \gamma$ is important for the determination of V_{ts} and as a window on new physics. Calculations here are hampered by the necessity to impose models for form factors. For the B meson decay constant, the systematic uncertainty arising from continuum extrapolation is being reduced and quenching effects are being addressed. Combining with results for the mixing parameter, the phenomenologically relevant quantity $f_B^2 B_B$ will become better determined.

2. SEMILEPTONIC AND RADIATIVE HEAVY-TO-LIGHT DECAYS

Lattice calculations of form factors are crucial for these decays: heavy quark symmetry relates different matrix elements and hence form factors, but the overall normalisation at the zero recoil point $\omega = v \cdot v' = 1$ is not fixed by the symmetry as it is for a heavy-to-heavy transition. Here v and v' are the four-velocities of the b quark and the light quark it decays into, respectively.

In reviewing results for these decays, I would like to emphasize how lattice calculations may be used for model independent extractions of CKM angles.

The diagram in Fig. 1 illustrates the problem in obtaining form factors for B decays over the full range of $q^2 = M^2 + m^2 - 2Mm\omega$, when a heavy meson of mass M decays to a light meson of mass m , transferring four-momentum q to the leptons or photon. Calculations using the conventional lattice Dirac equation (hereafter referred to as the “conventional” approach) are currently performed with heavy quark masses around the charm mass. The ini-

Table 1

Leading M dependence of form factors for heavy-to-light decays in the helicity basis. The dependence follows from heavy quark symmetry applied at *fixed* velocity transfer $\omega = v \cdot v'$. Note that only three of the four A_i form factors for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ are independent.

Form factor	t -channel exchange	Leading M dependence
$B \rightarrow \rho l \nu$		
V	1^-	$M^{1/2}$
A_1	1^+	$M^{-1/2}$
A_2	1^+	$M^{1/2}$
A_3	1^+	$M^{3/2}$
A_0	0^-	$M^{1/2}$
$B \rightarrow \pi l \nu$		
f^+	1^-	$M^{1/2}$
f^0	0^+	$M^{-1/2}$
$B \rightarrow K^* \gamma$		
T_1	1^-	$M^{1/2}$
T_2	1^+	$M^{-1/2}$

tial heavy meson is given 0 or 1 lattice units of three-momentum, while the light final meson can generally be given up to two lattice units of spatial momentum, allowing q^2 to be varied from $q_{\max}^2 = (M-m)^2$ (where $\omega = 1$) down to $q^2 < 0$ at the D scale. Heavy quark symmetry shows that the form factors scale simply with the heavy mass M for fixed ω . Table 1 shows the leading M dependences at fixed ω for the form factors in the helicity basis [1], where the J^P quantum numbers are fixed for the t -channel. These are multiplied by power series in $1/M$ to build up the full M dependence.

As Fig. 1 shows, fixed- ω scaling sweeps all the measured points to a region near q_{\max}^2 for B decays. The problem is then to extrapolate to cover the range back down to $q^2 = 0$. This is particularly acute for the radiative decay $\bar{B} \rightarrow K^* \gamma$ where the two-body fi-

*Plenary talk presented at Lattice 96, 14th International Symposium on Lattice Field Theory, St Louis, USA, June 1996.

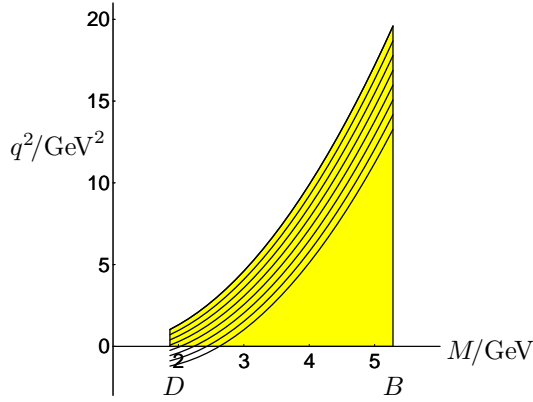


Figure 1. q^2 range for heavy-to-light decays as a function of the decaying heavy meson mass. Lines of constant ω are shown. The light final state mass is taken to be 850 MeV, typical of lattice pseudoscalar or vector meson masses before chiral extrapolation.

nal state with a real photon implies that only the form factors at $q^2 = 0$ contribute to the decay. Even when the heavy quark is treated using a static, non-relativistic or other modified action, the restriction on the available three-momenta on current lattices ensures that form factor values are obtained only near q_{max}^2 .

Some assistance is provided by ensuring that any model q^2 dependences employed satisfy the requirements of heavy quark symmetry together with any known constraints relating form factors at $q^2 = 0$. For example, f^+ and f^0 in $B \rightarrow \pi$ decays are related at $q^2 = 0$, and consistency is achieved by fitting f^+ to a dipole [pole] form and f^0 to a pole [constant] form. Heavy quark symmetry and light flavour $SU(3)$ symmetry relate the form factors for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ and $\bar{B} \rightarrow K^* \gamma$, and models can further relate these form factors to those for $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$. An overall fit to all the form factors might then be used. However, it is clearly desirable to avoid models entirely: I will concentrate on model independent results below.

Before discussing lattice results, I will summarise the experimental situation. The decays $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ and $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ are measured using full reconstruction by the CLEO collaboration, who find [2]:

$$B(\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l) = 1.8(4)(3)(2) \times 10^{-4}, \quad (1)$$

$$B(\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l) = 2.5(4)(\frac{5}{7})(5) \times 10^{-4}. \quad (2)$$

The experimental efficiency is model dependent, so the above results are a combined value using different models. When converted to a result for $|V_{ub}|$, the model dependence dominates the errors:

$$|V_{ub}| = 3.3(2)(\frac{3}{7})(7) \times 10^{-3},$$

to be compared with $|V_{ub}| = (3.1 \pm 0.8) \times 10^{-3}$ obtained from the lepton energy spectrum endpoint analysis in inclusive decays, which is also model dependent. The possibility of using lattice results to remove the model dependence from the determination of $|V_{ub}|$ is exciting motivation to pursue such studies.

The experimental result for $\bar{B} \rightarrow K^* \gamma$ also comes from CLEO, who quote [3],

$$B(\bar{B} \rightarrow K^* \gamma) = 4.2(8)(6) \times 10^{-5}. \quad (3)$$

Combining this with their measurement for the inclusive decay [4],

$$B(b \rightarrow s \gamma) = 2.32(57)(35) \times 10^{-4}, \quad (4)$$

they find [3],

$$R_{K^*} = \frac{\Gamma(\bar{B} \rightarrow K^* \gamma)}{\Gamma(b \rightarrow s \gamma)} = 0.181(68). \quad (5)$$

2.1. Semileptonic $B \rightarrow \pi$

Lattice results for the $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ form factors, f^+ and f^0 , are available from ELC [14], APE [15,16], UKQCD [10] and Wuppertal-HLRZ [12,13], together with preliminary results from FNAL [6,5]. They are plotted in Fig. 2 (the Wuppertal-HLRZ group extrapolated the form factors at $q^2 = 0$ only, using various ansätze, so their results are not displayed).

For massless leptons, the decay rate is determined by f^+ alone. However, the constraint $f^+(0) = f^0(0)$ (with suitable conventions) makes lattice measurements of both form factors useful. One procedure uses dispersive constraints to obtain model independent information, starting from the two-point correlation function of $V^\mu = \bar{u} \gamma^\mu b$,

$$\begin{aligned} \Pi^{\mu\nu}(q^2) &= i \int d^4x e^{iq \cdot x} \langle 0 | T V^\mu(x) V^{\nu\dagger}(0) | 0 \rangle \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_T(q^2) + q^\mu q^\nu \Pi_L(q^2). \end{aligned}$$

Because the contributions of intermediate B^* , $B\pi$, ... states to the corresponding spectral functions are all positive, the following inequality holds:

$$\text{Im } \Pi_L(t) \geq \phi(t) |f^0(t)|^2,$$

where $t = q^2$ and ϕ is a known function. A similar inequality relates $\text{Im } \Pi_T$ and f^+ . Combining the inequalities with subtracted dispersion relations for $\Pi_{T,L}$ provides bounds on the form factors in terms of quantities which can be evaluated in perturbative QCD.

Known values of the form factors can be incorporated to tighten the bounds. Lellouch [17] has shown how to incorporate the $f^+(0) = f^0(0)$ constraint together with imperfectly known values of the form factors, typical of lattice results with errors, to obtain

Table 2

Lattices used for calculations of heavy-to-light matrix elements. All simulations are quenched. In the FNAL case [5], heavy quarks are treated with the Fermilab formalism [6].

Label	Ref.	β	Lattice size	Cfgs	a^{-1}/GeV	Scale set by	Action
FNAL	[5]	5.9	$16^3 \times 32$	300	1.78	1P-1S in $c\bar{c}$	SW $c = 1.4$
BHSA	[7]	6.0	$16^3 \times 39$	39	2.10	f_π	W
BHSA	[7]	6.0	$24^3 \times 39$	16	2.29	f_π	W
LANL	[8]	6.0	$32^3 \times 64$	170	2.33	m_ρ	W
APE	[9]	6.0	$18^3 \times 64$	170	1.96	m_ρ	SW $c = 1$
UKQCDa	[10]	6.2	$24^3 \times 48$	60	2.73	$\sqrt{\sigma}$	SW $c = 1$
UKQCDB	[11]	6.2	$24^3 \times 48$	60	2.65	m_ρ	SW $c = 1$
BHSA	[7]	6.3	$24^3 \times 61$	20	3.01	f_π	W
WUP	[12,13]	6.3	$24^3 \times 64$	60	3.31	r_0	W
ELC	[14]	6.4	$24^3 \times 60$	20	3.7	m_ρ	W

families of bounds with varying confidence levels. A set of such bounds are shown in Fig. 3 together with the UKQCD [10] lattice measurements used to obtain them. In the figure, f^0 and f^+ are plotted back-to-back, showing the effect of imposing the constraint at $q^2 = 0$.

In Table 3 the bounds have been used to give ranges of values for the decay rate $\Gamma(\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l)$ in units of $|V_{ub}|^2 \text{ps}^{-1}$ together with values for the form factor f^+ at $q^2 = 0$. When combined with the experimental result for the decay rate in Eq. (1), one can extract $|V_{ub}|$ with about 35% theoretical error. Although this result is not very precise, the procedure used relies only on lattice calculations of matrix elements and heavy quark symmetry, together with perturbative QCD and analyticity properties in applying the dispersive constraints. There is no model dependence. Development and application of improved lattice actions for heavy quarks may in future remove the need for heavy quark symmetry in the extrapolation from D to B mesons. It is tempting to consider that lattice results could eventually supplant models in the experimental efficiency determinations, thereby removing model dependence from the experimental results.

Also shown in Table 3, for comparison, are values obtained from lattice calculations where assumed q^2 dependences have been imposed. For ELC [14] and APE [15], one value of f^+ has been used, at the given value of q^2 , fitted to a single pole form with pole mass m_p . The UKQCD result [10] is obtained from a combined dipole/pole fit to all measured f^+/f^0 points. Note that the UKQCD points have statistical errors only and have not been chirally extrapolated—they correspond to a pion mass of around 800 MeV (a similar caveat applies for the FNAL [6,5] results for f^+

Table 3

Results for $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ from dispersive constraints applied to lattice results [17], together with results obtained using ansätze for the form factor f^+ . Collaboration labels in the left hand column refer to lattices described in Table 2. The decay rates are values for the quantity $\Gamma(\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l)/|V_{ub}|^2 \text{ps}^{-1}$

	Rate	$f^+(0)$	
Dispersive	2.4–28	−0.26–0.92	95% CL
Constraint	3.6–17	0.00–0.68	70% CL
[17]	4.8–10	0.18–0.49	30% CL
ELC	9 ± 6	0.10–0.49	
$q^2 \simeq 18 \text{ GeV}^2$, pole fit, $m_p = 5.29(1) \text{ GeV}$ [14]			
APE	8 ± 4	0.23–0.43	
$q^2 \simeq 20.4 \text{ GeV}^2$, pole fit, $m_p = 5.32(1) \text{ GeV}$ [15]			
UKQCDa	7 ± 1	0.21–0.27	
dipole/pole fit to f^+/f^0 [10,18]			

and f^0). The results given [18] have used these values as though they applied to the physical pion. In obtaining bounds based on these points, Lellouch [17] added a conservatively estimated systematic error including terms to account for the chiral extrapolation (this error has been added to the UKQCD and FNAL points plotted in Fig. 2). Needless to say, improved lattice results can be used as input for the bounds once they become available.

Discretisation errors are an important source of systematic uncertainty [6]. Mass dependent errors affect the crucial extrapolation from D to B mesons in calculations using standard Wilson or

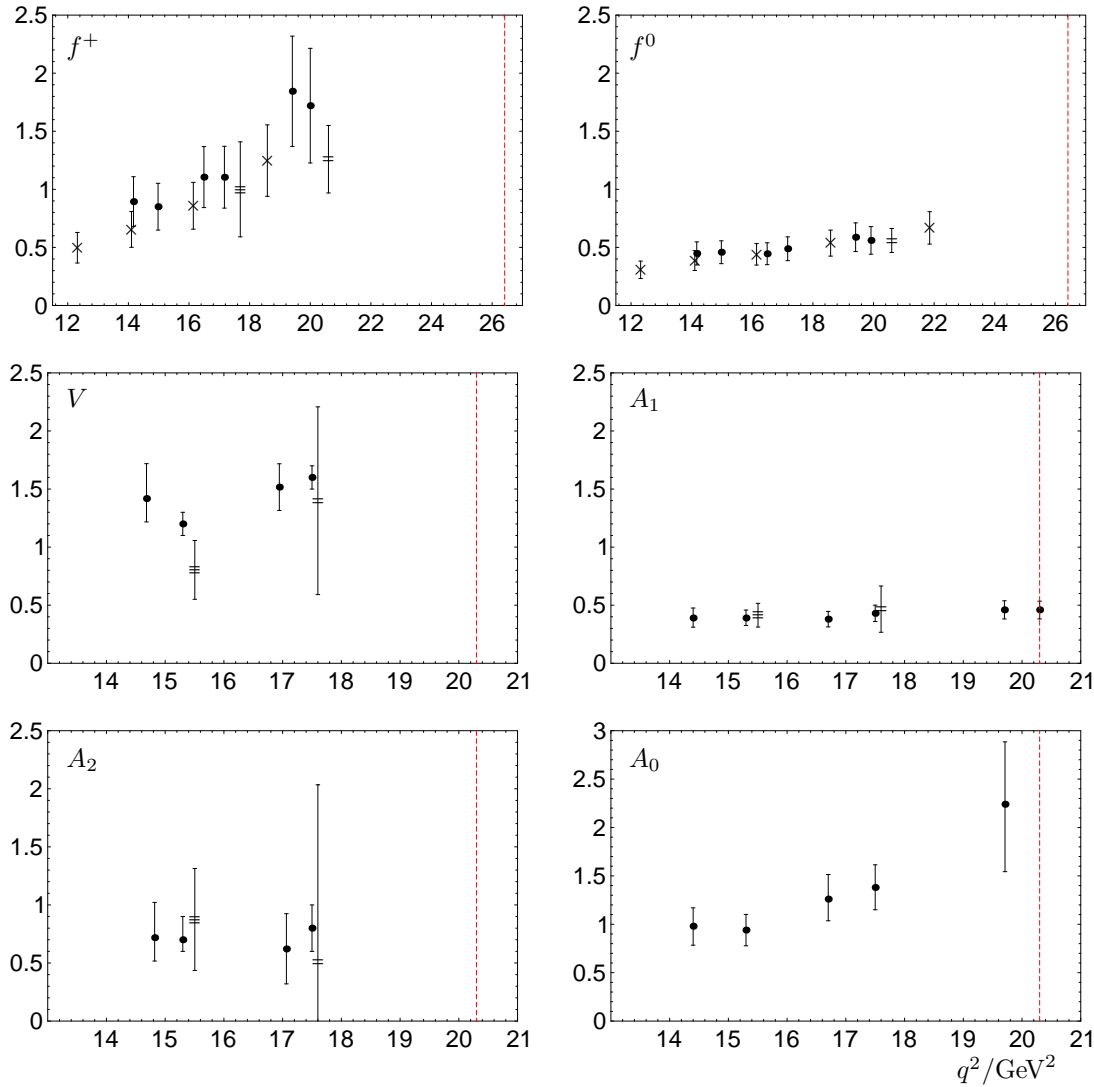


Figure 2. Lattice measurements of the form factors for $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ and $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$. Points are from ELC [14] (\equiv), APE [15,16] (\equiv), UKQCD [10] (\bullet) and FNAL [6,5] (\times , preliminary). Systematic errors have been added to the UKQCD and FNAL points, following [17]. The vertical dashed lines show q_{max}^2 .

Sheikholeslami-Wohlert (SW) quarks. Momentum dependent discretisation errors and increasing statistical uncertainty also limit the range of spatial momenta and hence q^2 that can be probed.

Discretisation errors in the heavy-mass scaling were studied by the FNAL group [6,5], using the Fermilab improvement formalism. They studied $\langle \pi | V^\mu | P \rangle / \sqrt{2ME_\pi}$, where P is a pseudoscalar meson of mass M . This quantity should be constant in the $M \rightarrow \infty$ limit. The matrix elements were computed for D and B mesons as well as in the static limit. The improvement prescription resulted in little dependence on $1/M$ for a range of pion momenta.

Momentum-dependent discretisation errors have been studied by a FNAL-Illinois-Hiroshima

group [20] also using the Fermilab formalism for heavy quarks. From a study of the axial current matrix element between a pseudoscalar and the vacuum, they estimate that momentum dependent errors are less than 20% for light mesons with $|\mathbf{p}| < 1.2 \text{ GeV}$ on a relatively coarse $\beta = 5.7$ lattice.

Improvement will clearly help in the determination of reliable phenomenological results from lattice calculations. It will be particularly interesting to test the benefit of the $O(a)$ improvement program outlined by the ALPHA collaboration [21].

I note finally in this section that chiral extrapolations are severe for the $B \rightarrow \pi$ matrix elements. As the pion mass approaches its physical value, the B^* -pole and the beginning of the $B\pi$ continuum move

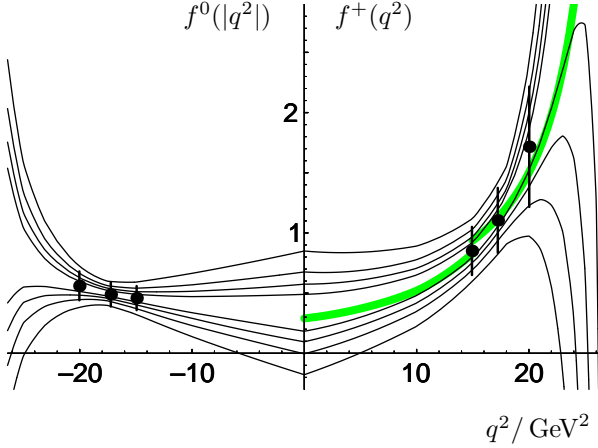


Figure 3. Bounds on the form factors f^+ and f^0 for $\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu}_l$ obtained from dispersive constraints [17]. The points displayed are from UKQCD [10], with added systematic errors, and the shaded band is the prediction of a light-cone sum rule calculation [19].

very near the upper endpoint of q^2 . The form factors may vary rapidly with the pion mass near q_{\max}^2 . For this reason, the lattice calculations of semileptonic $B \rightarrow \rho$ decay are currently most reliable and I now turn to this.

2.2. Semileptonic $B \rightarrow \rho$

To avoid models for the q^2 dependence of the form factors, we use the lattice results directly. The lattice can give the differential decay rate $d\Gamma/dq^2$, or the partially integrated rate, in a q^2 region near q_{\max}^2 up to the unknown factor $|V_{ub}|^2$. For example, UKQCD [11] parametrised the differential decay rate near q_{\max}^2 by,

$$\frac{d\Gamma}{dq^2} = \text{const} |V_{ub}|^2 q^2 \lambda^{1/2} a^2 (1 + b[q^2 - q_{\max}^2]), \quad (6)$$

where λ is the usual phase space factor and a and b are constants. The constant a plays the role of the Isgur-Wise function evaluated at $\omega = 1$ for heavy-to-heavy transitions, but in this case there is no symmetry to determine its value at leading order in the heavy quark effective theory.

For massless leptons, the differential decay rate depends on the V , A_1 and A_2 form factors, but A_1 is the dominant contribution near q_{\max}^2 and is also the best measured on the lattice, as shown in Fig. 2.

Fig. 4 shows UKQCD results for the differential decay rate together with a fit to the parametrisation in Eq. (6). Because of phase space suppression, the point at q_{\max}^2 does not influence the fit, and so gives another determination of a^2 . The values found

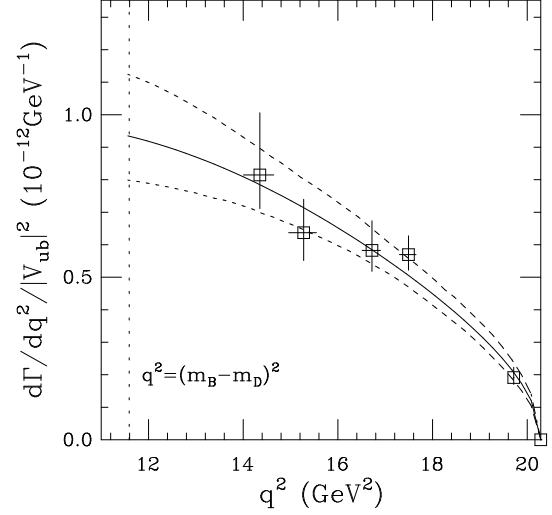


Figure 4. UKQCD calculation of the differential decay rate for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ as a function of q^2 [11].

are [11],

$$a^2 = \begin{cases} 21(3) \text{ GeV}^2 & \text{fit to 5 points,} \\ 23(2) \text{ GeV}^2 & q_{\max}^2 \text{ point,} \end{cases}$$

leading to:

$$a = 4.6_{-0.4}^{+0.3}(\text{stat}) \pm 0.6(\text{syst}) \text{ GeV}.$$

Discounting experimental errors, this result will allow determination of $|V_{ub}|$ with a theoretical uncertainty of 10% statistical and 12% systematic. With full reconstruction of events, CLEO are beginning to extract the differential decay distributions [2]. The Babar experiment, with unequal beam energies allowing verification that the ρ and lepton originate from the same vertex, should do even better.

The UKQCD results for V , A_1 and A_2 agree very well with a light cone sum rule (LCSR) calculation of Ball and Braun [22]. The agreement is perhaps fortuitous. More interestingly, LCSR calculations predict that all the form factors for heavy-to-light decays have the following heavy mass dependence at $q^2 = 0$ [19,23]:

$$f(0) = \frac{1}{M^{3/2}} (a_0 + a_1/M + a_2/M^2 + \dots). \quad (7)$$

The leading M dependence comprises \sqrt{M} from the heavy state normalisation together with the behaviour of the leading twist light cone wavefunction. For $A_1(0)$ the LCSR result is [22] $A_1(0) \simeq 0.26$. UKQCD fitted the heavy mass dependence of $A_1(0)$ to the form in Eq. 7 and found [11]

$$A_1(0) = \begin{cases} 0.18 \pm 0.02 & a_0 \text{ and } a_1 \neq 0 \\ 0.22_{-0.03}^{+0.04} & a_0, a_1 \text{ and } a_2 \neq 0 \end{cases}$$

Table 4

Lattice results for $\bar{B} \rightarrow K^* \gamma$. The labels in the left hand column refer to the lattices described in Table 2.

	$T(0)$		$T_2(q_{\max}^2)$
	$M^{-3/2}$	$M^{-1/2}$	
BHSb	0.10(3)		0.33(7)
LANL	0.09(1)	0.24(1)	
APE	0.09(1)(1)	0.23(2)(2)	0.23(2)(2)
UKQCDa	0.15($^{7}_{-6}$)	0.26($^{2}_{-1}$)	0.27($^{2}_{-1}$)
BHSc			0.30(3)

Pole fits for A_1 (and dipole/pole for f^+/f^0 for $B \rightarrow \pi$ decays) have leading $M^{-3/2}$ behaviour at $q^2 = 0$, so we can also compare with other lattice results:

$$A_1(0) = \begin{cases} 0.22 \pm 0.05 & \text{ELC [14]} \\ 0.24 \pm 0.12 & \text{APE [15]} \\ 0.27^{+7}_{-4} & \text{UKQCD [11]} \end{cases}$$

The Wuppertal-HLRZ group [12,13] tried various forms for the heavy mass dependence of $A_1(0)$, although none had a leading $M^{-3/2}$ dependence.

2.3. Rare radiative $B \rightarrow K^* \gamma$

This decay was discussed in some detail by A. Soni at Lattice 95 [24] so my comments will be brief. In Table 4 I summarise the available lattice results, all from quenched simulations for the matrix element

$$\langle K^*(k, \eta) | \bar{s} \sigma_{\mu\nu} q^\nu b_R | B(p) \rangle \quad (8)$$

which is parameterised by three form factors, T_i , $i = 1, 2, 3$. For the decay rate the related values $T_1(0)$ and $T_2(0)$ are needed. Suitably defined, they are equal, so in the table I quote a single value $T(0)$, together with the directly measured $T_2(q_{\max}^2)$. The results are classified according to the leading M dependence of the form factor at $q^2 = 0$ which is governed by the model used to fit the q^2 dependence. Dipole/pole forms for T_1/T_2 give $M^{-3/2}$ behaviour and pole/constant forms give $M^{-1/2}$. The table shows that the results agree when the same assumptions are made. All groups find that T_2 has much less q^2 dependence than T_1 , but the overall forms cannot be decided, so a phenomenological prediction is elusive.

Additional long distance contributions may not be negligible so the matrix element of Eq. (8) may not give the true decay rate [25–27]. Once the q^2 dependence of the form factors is known, lattice calculations of the ratio $R_{K^*} = \Gamma(\bar{B} \rightarrow K^* \gamma) / \Gamma(b \rightarrow s \gamma)$ can be compared to the experimental result in Eq. (5) to test for long distance effects.

Heavy quark symmetry, combined with light flavour $SU(3)$ symmetry, relates the form factors for $\bar{B}^0 \rightarrow \rho^+ l^- \bar{\nu}_l$ and $\bar{B} \rightarrow K^* \gamma$ [28,29] in the infinite heavy quark mass limit. On the lattice these relations can be tested using identical light quarks. UKQCD [11] showed that the ratios $V/2T_1$ and $A_1/2T_2$ both satisfied the heavy quark symmetry prediction of unity in the infinite mass limit. A combined fit of the pseudoscalar to vector form factors consistent with heavy quark symmetry could help resolve the ambiguity in the q^2 dependence of the $\bar{B} \rightarrow K^* \gamma$ form factors.

3. LEPTONIC DECAY CONSTANTS OF HEAVY-LIGHT PSEUDOSCALARS

Leptonic decay constants of heavy light systems were fully reviewed by C. Allton at Lattice 95 [30], so here I report new results. To establish notation, the decay constant is determined by lattice calculation of the dimensionless quantity Z_L according to,

$$f_P \sqrt{M_P/2} = Z^{\text{ren}} Z_L a^{-3/2}$$

where Z^{ren} is the renormalisation constant required to match to the continuum and a is the lattice spacing.

3.1. Conventional Methods

New results were shown by the MILC [31] and JLQCD [32] collaborations. Both groups perform continuum extrapolations from results at several β 's.

MILC have six sets of quenched configurations for β in the range 5.7–6.52, together with six sets of two-flavour dynamical staggered fermion configurations with $5.445 \leq \beta \leq 5.7$. A Wilson action is used for the valence quarks, with a hopping parameter expansion [33] for the heavy quark for easy simulation of a range of masses. The heavy quark has a Kronfeld-Lepage-Mackenzie normalisation, $\sqrt{1 - 6\kappa}$, together with a shift from the pole mass to the kinetic mass for the heavy meson. The continuum extrapolation should deal with remaining $O(a)$ errors. Fig. 5 shows this extrapolation for f_B . MILC's (preliminary) results are:

$$\begin{aligned} f_B &= 166(11)(28)(14) \text{ MeV} \\ f_{B_s} &= 181(10)(36)(18) \text{ MeV} \\ f_{B_s}/f_B &= 1.10(2)(5)(7) \\ f_D &= 196(9)(14)(8) \text{ MeV} \\ f_{D_s} &= 211(7)(25)(11) \text{ MeV} \\ f_{D_s}/f_D &= 1.09(2)(5)(5) \end{aligned}$$

The central values come from the quenched results using a linear extrapolation in a with the scale set by f_π and linear chiral extrapolations (in $1/\kappa$). The

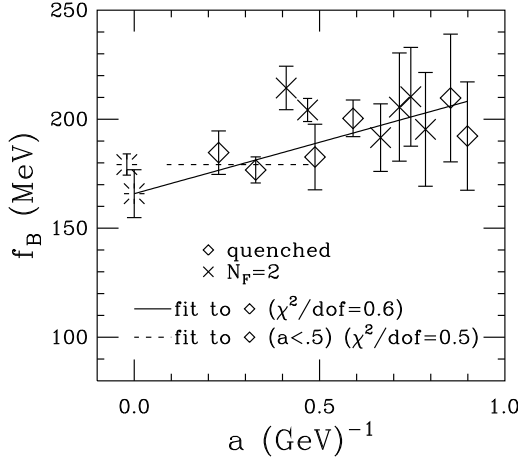


Figure 5. Continuum extrapolation of f_B by MILC [31]. The dashed line is a constant fit for quenched results with $\beta \geq 6.0$.

heavy mass dependence is determined by a fit for heavy meson masses in the range $1.5 \text{ GeV} < M < 4 \text{ GeV}$ combined with a point in the static limit. The first error quoted is a combination of the statistical error and that coming from choice of fitting procedure, the second error is remaining systematic errors within the quenched approximation and the final error is for quenching.

Improved covariant fitting methods have changed the results compared to their values at Lattice 95 [34], although within errors. In particular the difference between quenched and unquenched results is less dramatic, but still looks significant (especially for f_{B_s}), and indicates an increase in the value of the decay constant for full QCD.

JLQCD [32] have results from three β values, 5.9, 6.1 and 6.3 with quenched configurations using the Wilson action for both light and heavy quarks. They study different prescriptions for reducing the $O(ma)$ scaling violations associated with the heavy quark and aim to show that results from all prescriptions converge in the continuum limit. Smeared wavefunctions are determined for each available combination of heavy and light quark kappa values. Results are quoted using the charmonium 1S–1P splitting to set the scale.

JLQCD apply four different procedures to extract the decay constant. The first uses the traditional $\sqrt{2\kappa}$ quark field normalisation and the meson pole mass. Three further methods use a Kronfeld-Lepage-Mackenzie normalisation combined with, (a) setting the meson mass from the pole mass, (b) pole mass shifted to kinetic mass at tree level, or (c) measured kinetic mass. For all four methods, a linear extrapo-

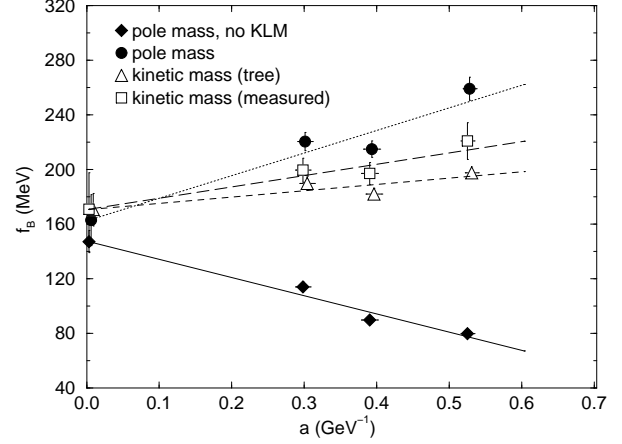


Figure 6. Continuum extrapolation of f_B by JLQCD [32] using different prescriptions.

lation in a gives the continuum result. This is shown for f_B in Fig. 6. The preliminary results are [32]:

$$\begin{aligned} f_B &= 163(19)_{(16)}^{(8)} \text{ MeV} \\ f_{B_s} &= 174(12)_{(19)}^{(17)} \text{ MeV} \\ f_D &= 191(8)_{(8)}^{(4)} \text{ MeV} \\ f_{D_s} &= 210(6)_{(12)}^{(6)} \text{ MeV} \end{aligned}$$

where the first error is statistical and the second is from the spread over the four methods above.

The MILC and JLQCD results are included in Table 5 which summarises determinations of heavy-light pseudoscalar meson decay constants and their ratios. MILC and JLQCD are (currently) in pleasing agreement. From the table, and allowing for an increase in the value from unquenching, I quote a global result for f_B :

$$f_B = 175 \pm 25 \text{ MeV}.$$

3.2. Decay Constants from NRQCD

The SGO collaboration have calculated matrix elements for determining pseudoscalar and vector B meson decay constants [45,46] using both Wilson and tadpole-improved SW light quarks combined with NRQCD heavy quarks. They have 100 unquenched configurations with $n_f = 2$ flavours of dynamical staggered quark ($m_{\text{sea}}a = 0.01$) at $\beta = 5.6$. The NRQCD action is corrected to $O(1/M)$ and matrix elements are calculated for extra operators appearing at $O(1/M)$ in the matching of NRQCD currents onto full QCD vector and axial vector currents².

Appropriate renormalisation constants are not yet available, however, for the matching to full QCD, so I

²See Refs. [45,46] for details and qualifications.

Table 5

Summary of results for values and ratios of leptonic decay constants of pseudoscalar mesons. Statistical and systematic errors have been combined in quadrature. The results from MILC [31] include a systematic error for quenching. Results from APE [35] and FNAL [5] show statistical errors only. FNAL [5] results are preliminary and use a nominal value for the strange quark mass to determine f_{B_s} and f_{D_s} . Results with no quoted values for β or a^{-1} come from more than one lattice: PSI-WUP [36], MILC [31] and JLQCD [32] perform a continuum extrapolation using their own data.

	Ref	β	$\frac{a^{-1}}{\text{GeV}}$	$\frac{f_B}{\text{MeV}}$	$\frac{f_{B_s}}{\text{MeV}}$	$\frac{f_{B_s}}{f_B}$	$\frac{f_D}{\text{MeV}}$	$\frac{f_{D_s}}{\text{MeV}}$	$\frac{f_{D_s}}{f_D}$
FNAL	[5]	5.9		188 ⁽⁶⁾ ₍₄₎	207 ⁽³⁾ ₍₂₎		220 ⁽⁴⁾ ₍₅₎	239 ⁽³⁾ ₍₄₎	
DeG-L	[37]	6.0	1.9				190(33)	222(16)	1.17(22)
APE	[35]	6.0		197(18)			218(9)	240(9)	1.11(1)
UKQCD	[38]	6.0	2.0	176 ⁽⁴¹⁾ ₍₂₈₎		1.17(12)	199 ⁽³⁰⁾ ₍₂₄₎		1.13 ⁽⁶⁾ ₍₇₎
LANL	[39]	6.0	2.33				229 ⁽²¹⁾ ₍₁₇₎	260 ⁽²⁷⁾ ₍₂₂₎	1.14(2)
BDHS	[40]	6.1	2				174(53)	234(72)	1.35(22)
UKQCD	[38]	6.2	2.7	160 ⁽⁵³⁾ ₍₂₀₎	194 ⁽⁶²⁾ ₍₁₀₎	1.22 ⁽⁴⁾ ₍₃₎	185 ⁽⁴²⁾ ₍₈₎	212 ⁽⁴⁶⁾ ₍₈₎	1.18(2)
BLS	[41]	6.3	3.0	187(38)	207(41)	1.11(6)	208(38)	230(36)	1.11(6)
ELC	[42]	6.4	3.3				210(40)	230(50)	
ELC	[43]						194(15)		
ELC	[42]			205(40)		1.08(6)			
PSI-WUP	[44]						198(17)	209(18)	
LANL	[39]						186(29)	218(15)	
PSI-WUP	[36]		$a \rightarrow 0$	180(50)		1.09(5)	170(30)		1.09(5)
MILC	[31]		$a \rightarrow 0$	166(33)	181(41)	1.10(9)	196(18)	211(28)	1.09(7)
JLQCD	[32]		$a \rightarrow 0$	163 ⁽²¹⁾ ₍₂₅₎	174 ⁽²¹⁾ ₍₂₂₎		191 ⁽⁹⁾ ₍₁₁₎	210 ⁽⁸⁾ ₍₁₃₎	

do not quote here any values for decay constants. Accepting this limitation, the slope of the pseudoscalar decay constant with respect to $1/M$ is calculated, that is, the term c_P in the expansion

$$f\sqrt{M} = \text{const}(1 + c_P/M + \dots).$$

The results are,

$$c_P = \begin{cases} -1.35(15)a^{-1} = -2.8(5) \text{ GeV} & \text{Wilson,} \\ -1.0(2)a^{-1} \sim -2 \text{ GeV} & \text{TI SW,} \end{cases}$$

to be compared to slopes of around 1 GeV found using conventional methods. Study of a spin average of pseudoscalar and (appropriately defined) vector meson decay constants shows that just over 90% of the effect comes from the heavy quark kinetic energy term. Analysing the effect of the kinetic term on wavefunctions and energies in the static limit (from a spinless relativistic quark model) to first order shows that a slope of 1–2 GeV is natural, and also predicts that the 1S state energy should rise slowly with $1/M$, with the 2S energy rising faster [47].

It will be very interesting to see the slope and decay constant values once the correct renormalisation constants are included.

3.3. f_B^{stat} from Bermions

The APETOV group have results for f_B^{stat} obtained using a pseudofermion method [48]. In standard static quark methods, smeared sources must be tuned to optimise the projection onto the ground state and avoid noise. Moreover, the numerical expense of inverting the Dirac operator restricts the set of available light quark propagators to have one endpoint fixed. In contrast, APETOV determine the light propagator by Monte Carlo inversion, allowing them to average the static correlator over *all* points without requiring smeared sources. The additional average over lattice points helps overcome the extra noise inherent in the Monte Carlo inversion. Further averaging over the gauge links is also employed.

The bermion action is $\sum_x |Q\phi(x)|^2$, where x is a lattice point, U is the gauge field, ϕ the scalar bermion field and Q is given by $Q\phi(x) = \gamma_5 D\phi(x)$, where D is the usual Wilson lattice Dirac operator. The bermions are thermalised in each gauge configuration and the pseudofermion two-point function then gives $(Q^2)^{-1}$. Remultiplying by Q and γ_5 allows the usual light quark Dirac propagator to be determined between any lattice points. The correlator of two local heavy-light bilinears with a given

time separation is then averaged over all points to determine Z_L . In practice, a further correlator using a $(Q^2)^{-1}$ fermion propagator with the static propagator, which better isolates the lowest lying state, needs to be analysed concurrently.

The method has been applied to two $16^3 \times 32$ lattices with 30 gauge configurations each at $\beta = 5.7$ and 6.0, with the results [49]:

$$Z_L = \begin{cases} \begin{cases} 0.477(31) & \kappa_{\text{crit}} \\ 0.582(11) & \kappa_{\text{strange}} \end{cases} & \beta = 5.7 \\ \begin{cases} 0.188(37) & \kappa_{\text{crit}} \\ 0.216(13) & \kappa_{\text{strange}} \end{cases} & \beta = 6.0 \end{cases}$$

The $\beta = 6.0$ chirally extrapolated result agrees well with world results for Z_L . At $\beta = 5.7$ the result is below the FNAL [50] value $Z_L = 0.564(28)$, but is on a larger lattice and has comparable errors with one third the number of gauge configurations.

APETOV have extended the method by making the fermions dynamical [51]. Each fermion flavour corresponds to -2 fermion flavours. Z_L is calculated for different $n_b = -2n_f$, tuning β to keep the ratio $R = m_\pi^2/m_\rho^2$ constant, starting with $\beta = 5.7$ for $n_b = 0$. Including a one-loop perturbative value for Z^{ren} , the result for the ratio of $Z_L Z^{\text{ren}}$ for 3 flavours to 0 flavors is 1.14(2) for $R = 0.6$ and 1.16(4) for $R = 0.5$, with the effect increasing to about 20% in the chiral limit. This offers more evidence for the increase of f_B once dynamical effects are included.

4. Results for B Mixing

Recent results for the B meson mixing parameter, $B_B(\mu) = \langle \bar{B} | \mathcal{O}(\mu) | B \rangle / (8/3) f_B^2 M_B^2$, where $\mathcal{O} = b\gamma_\mu(1-\gamma_5)q\bar{b}\gamma^\mu(1-\gamma_5)q$ is the $\Delta B = 2$ operator and M_B is the B meson mass come from UKQCD [54], APE [53] and the Kentucky group [52] in the static limit, together with results [24,55] using conventional methods. The APE result makes use of a new calculation [58] of the full-theory/static-theory matching which incorporates previously omitted contributions. Table 6 is a compilation of lattice determinations of B_B . To ease comparison, values are given for $B_B(m_b=5 \text{ GeV})$ and for the (1-loop) renormalisation group invariant quantity, $\hat{B}_B = \alpha_s(\mu)^{-2/\beta_0} B_B(\mu)$. A similar collection of results appears in the contribution by Christensen, Draper and McNeile [52] to these proceedings.

The conventional results are consistent and show no a dependence. The static results show substantial differences arising from the renormalisation constants. The B_B calculation involves dividing by the square of Z_A , the heavy-light axial current renormalisation, which differs for Wilson (KEN) and SW (UKQCD, APE) light quarks. Moreover, whether

products of renormalisation constants are expanded to a given order in α or simply multiplied makes a significant difference: the UKQCD result for \hat{B}_B rises to 1.19(6) [54] when the factors are multiplied rather than expanded. Nonperturbative determinations of the renormalisation factors are clearly crucial to reduce systematic errors [59,60].

The relevant phenomenological quantity is $f_B^2 B_B$ which can be extracted directly from the $\Delta B = 2$ matrix element. To avoid uncertainties from setting the scale, it is convenient to determine the ratio $f_{B_s}^2 B_{B_s} / f_B^2 B_B$. B_{B_s} / B_B is found to be close to unity in recent calculations: 1.01(3)(3) [24] and 1.011(8) [53]. For f_{B_s} / f_B , the values in Table 5 can be combined with static results,

$$\frac{f_{B_s}^{\text{stat}}}{f_B^{\text{stat}}} = \begin{cases} 1.11(2) & [41] \\ 1.13^{(4)}_3 & [38] \\ 1.22(4)(2) & [50] \\ 1.16^{(4)}_3 & [54] \\ 1.17(3) & [53,61] \end{cases}$$

The second error in the first result for B_{B_s} / B_B is for quenching based on numerical evidence for a small increase in the ratio on $n_f = 2$ dynamical configurations (Sharpe and Zhang [62,63] estimate a quenching error of -0.04 based on chiral loops). Unquenching is expected to increase the value of f_{B_s} / f_B by about 10% (the chiral loop estimate in 0.16 [62,63]).

Bernard, Blum and Soni [55,64] report a preliminary value for

$$r_{sd} = \frac{m_{B_s}^2 f_{B_s}^2 B_{B_s}}{m_B^2 f_B^2 B_B} = 1.81(8)(25).$$

The ratio shows no evidence of a dependence on a range of lattice spacings. The result implies rather a large value for f_{B_s} / f_B , compared to other lattice results: it will be interesting to see the value of this ratio when the decay constants are extracted separately from the same data.

5. OTHER RESULTS

Heavy-to-heavy quark transitions have been omitted from this report. See [65] for a recent compilation of lattice results for the Isgur-Wise function relevant for $B \rightarrow D^{(*)}$ semileptonic decays. UKQCD presented preliminary results at this conference for the baryonic Isgur-Wise function in semileptonic $\Lambda_b \rightarrow \Lambda_c$ decays [66].

Results for λ_2 , the matrix element of the chromomagnetic moment operator between heavy mesons in the heavy quark effective theory, continue to sit at about half the experimental value [53,54]. The one loop renormalisation constant is uncomfortably

Table 6

Lattice results for the B_d meson mixing parameter B_B . Lattice parameters are quoted except where the authors have performed a continuum extrapolation from several results, indicated by $a \rightarrow 0$ in the a^{-1} column. The column labelled “Action” shows the action used for the light and heavy quarks respectively. The b quark mass m_b has been set to 5 GeV: where a result has been quoted at a different scale, the scale μ and $B_B(\mu)$ have been listed. The results headed “Static fitted” are obtained by extrapolating $1/M \rightarrow 0$ where M is the heavy-light meson mass. Results in oblique type have been obtained from the authors’ values using one-loop scaling with 5 flavours and $\Lambda_{(5)\overline{\text{MS}}} = 130$ MeV.

	Ref	β		Cfgs	$\frac{a^{-1}}{\text{GeV}}$	Action	$\frac{\mu}{\text{GeV}}$	$B_B(\mu)$	$B_B(m_b)$	\hat{B}_B
Static										
KEN	[52]	6.0	$20^3 \times 30$	32	2.1	W-stat	4.33	0.98(5)	0.97(5)	1.45(8)
APE	[53]	6.0	$24^3 \times 40$	600	2.0	SW-stat			0.82(4)	1.21(6)
UKQCD	[54]	6.2	$24^3 \times 48$	60	2.9	SW-stat			0.69(4)	1.02(6)
Static fitted										
ELC	[42]	6.4	$24^3 \times 60$	20	3.7	W-W	3.7	0.90(5)	0.88(5)	1.30(7)
BS	[24]	5.7–6.3			$a \rightarrow 0$	W-W	2	1.04(5)	0.96(5)	1.42(7)
Conventional										
BBS	[55,56]	5.7	$16^3 \times 33$	100	1.45	W-W	2	0.96(5)	0.89(5)	1.32(7)
BBS	[55,56]	6.0	$16^3 \times 39$	60	2.06	W-W	2	0.96(5)	0.89(5)	1.32(7)
BBS	[55,56]	6.0	$24^3 \times 39$	40	2.22	W-W	2	0.98(4)	0.90(3)	1.34(5)
JLQCD	[57]	6.1	$24^3 \times 64$	200	2.56	W-W			0.90(5)	1.32(7)
JLQCD	[57]	6.3	$32^3 \times 80$	100	3.38	W-W			0.84(6)	1.24(9)
BBS	[55,56]	6.3	$24^3 \times 61$	60	3.40	W-W	2	1.09(16)	1.01(15)	1.49(22)
ELC	[42]	6.4	$24^3 \times 60$	20	3.7	W-W	3.7	0.86(5)	0.84(5)	1.24(7)
BDHS	[40]	5.7–6.1			$a \rightarrow 0$	W-W	2	1.10(15)	0.94(14)	1.38(21)
BS	[24]	5.7–6.3			$a \rightarrow 0$	W-W	2	0.96(6)	0.89(6)	1.31(8)

large: this and other systematic effects warrant further study given the importance of λ_2 in inclusive B decays.

Semileptonic $D \rightarrow K^{(*)}$ decays were reviewed by J. Simone at Lattice 95 [6].

Acknowledgement

I am very grateful to the following for supplying information about their results: Arifa Ali Khan, Claude Bernard, Tom Blum, Joseph Christensen, Sara Collins, Christine Davies, Giulia de Divitiis, Vincent Giménez, Shoji Hashimoto, Laurent Lellouch, Vittorio Lubicz, Craig McNeile, Jim Simone and Nicoletta Stella. I have also received valuable help from Patricia Ball, Aida El Khadra, Andreas Kronfeld, Guido Martinelli, Juan Nieves, Chris Sachrajda, Amarjit Soni and Hartmut Wittig. Where I have received new or updated results since the St. Louis conference, I have endeavoured to include them in this report, but some quoted results may already be out of date.

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